STATISTICS

Paper 4040/12

Paper 12

Key Messages

After performing a calculation which relates to a specific physical situation, it is worth pausing to consider whether or not the answer obtained is a sensible one. If it is not, the work should be checked to find the error.

Candidates should always try to relate their knowledge to the specific requirements of a question, including the specific context involved.

If a question specifies a certain degree of accuracy for numerical answers, full marks will not be obtained if the instruction is not followed.

General Comments

The overall standard of work was very good this year. A substantial number of candidates obtained high marks, and there were few exceptionally low marks. It has been noted regularly in these reports that marks are often lost due to final answers not being given to the accuracy specifically stated in the question. Some candidates clearly show awareness of this, and follow any accuracy instructions, but others still do not.

It has also been pointed out regularly that a candidate of statistics ought to be able to observe whether or not the result of a calculation is reasonable for the practical situation of the question. If it is clearly unreasonable, the work leading to it should be checked to find the error. But some candidates seem to treat the work they do as exercises in pure mathematics, having no practical relevance. For example, in a simple experiment involving the cooling of hot water (see **Question 7** below) it should have been obvious that, from a temperature of 38°C, the temperature could not possibly, in the next five minutes, rise far above the boiling point of water.

To questions requiring a written answer there is sometimes a tendency for answers to be presented which are effectively answers to other (old) questions from previous examinations. This happens especially with questions on crude and standardised rates (see **Question 11(vi)** below). Written answers should always refer to the specific context of the given question.

Comments on Specific Questions

Section A

Question 1

The differences between these sampling methods were generally well understood.

Answers: **(i)** systematic **(ii)** quota **(iii)** stratified

Question 2

Most candidates knew which statistical measures are measures of central tendency, and which are measures of dispersion. As the question repeated the instruction "name" in every part, abbreviations such as "IQR", which may commonly be used in classwork, were not accepted. Such abbreviations do not show the candidate's knowledge of the *name* of the measure.

A common error was seen in part **(iv)** in finding the interquartile range. The given distribution consists of ungrouped data, and as such the positions of the quartiles are found from cumulative frequencies of $(n + 1)/4$ and $3(n + 1)/4$, not from the $n/4$ and $3n/4$ which were commonly seen.

Answers: **(i)** mean (or mode) **(ii)** median **(iii)** standard deviation (or variance or range)
(iv) interquartile range, 3

Question 3

Most of the answers seen to part **(i)** were fully correct. In part **(ii)** a few made the inappropriate comparison across the genders of the numbers observed choosing the different drinks; but most made the necessary comparison, for the claim made, of drinks preferences within genders. Thus very many answers were seen earning five marks. Almost all answers lost a mark through failure to point out the extreme limitations of the survey, and therefore the limitations on any conclusion reached, for investigating a claim containing (twice in the question) the word "generally". Only twenty people were surveyed, and even these were from a very narrow personal circle of acquaintances.

Answers: **(i)** two-way table with headed columns and rows, and cell values of 2, 5, 1, 3, 7, 2, in correct places **(ii)** from these data, for males no, for females yes; but the sample is far too small and limited for any general conclusion to be drawn

Question 4

In part **(i)** a common error in **(b)** was to give two shops, displaying lack of understanding of what the standard deviation measures, and also indicating that the words of the question, "...which one of the shops...", had not been carefully read. But very many fully correct answers to part **(ii)** were seen.

Answers: **(i)(a)** B **(b)** C **(c)** C **(ii)** 142

Question 5

There were mixed answers to part **(i)** where some candidates showed they understood clearly the rules of the game, and others did not.

Some fully correct, well presented answers to part **(ii)** were seen, but a fairly common error was to omit the case of 4 on all three dice. Candidates would be advised in such a question as this to set out their work in such a way that the different cases being considered, and their associated probabilities, can be seen clearly. Examiners may then award method marks when a final answer is incorrect. If many unattributed probabilities are seen (in such as case as this consisting of many 1/6 s) Examiners are unable to assess understanding, and therefore unable to award marks.

Answers: **(i)** Any four from 1, 5, 7, 11, 13, 14, 16, 17 **(ii)** 2/27

Question 6

With the exception of part **(i)**, this question was very well done. Many clearly set out solutions were seen to part **(iv)**.

Answers: **(i)** cumulative frequency polygon **(ii)** 2.8 **(iii)** 6.6 **(iv)** $552

Section B

Question 7

The way in which semi-averages are used to find a line of best fit to experimental or survey data is well known. But there is still some regular misunderstanding of how the semi averages are found in the first place. As was pointed out in this report last year, some candidates do not seem to appreciate that the pairs of x and y values given in such a question as this are linked, and instead treat them as though they are totally independent of each other. It was not unusual to see the very serious error made of taking the lowest x values with the lowest y values, and the highest x values with the highest y values. Such an error should have been obvious to the candidate subsequently when the gradient of the equation found turned out to be positive, but no indication of this recognition was ever seen.

Sometimes as a consequence of working with an incorrect equation from part **(ii)** in part **(iii)** the predicted temperature after five more minutes had an impossible value. Impossibly large values, both positive and negative were seen here, when it should have been obvious, from inspecting the table of data, that the temperature ought to be a few degrees lower than 38°C. When a practically impossible value is obtained (in any question) the thoughtful candidate should pause, knowing that the answer cannot be correct, and look back in the work to try to find the error. There were no instances of this happening here, though in a very few cases the candidate seemed to know that something was wrong and abandoned the question in favour of another.

For parts **(vi)** and **(vii)**, few candidates appear to have noticed that the plotted points clearly follow a curve.

Answers: **(i)** (12.5, 60.3), (5, 77.7), (20, 43) **(ii)** $y = -2.31x + 89.2$ **(iii)** 20°C **(vi)** the plotted points show that the relationship between these variables is not linear **(vii)** it will be higher than that calculated

Question 8

This question was well answered with candidates displaying good ability in interpreting statistical information presented in mixed tabular and pictorial form. In part **(iv)** the histogram was often fully correct. But those not understanding the principle on which histograms are constructed should have been aware that something was wrong when trying to draw a rectangle of height 72 units on a given grid extending to only 70 units.

There were many fully correct answers to parts **(v)**, **(vi)** and **(vii)**, although in part **(vii)** some did not use all the mature candidates in the calculation, but only those in Technology.

Answers: **(i)** 748 **(ii)** (525/1614) × 100 **(iii)** 12.6% **(iv)** rectangles with widths of 2, 3 and 5 years, and heights of 33, 24 and 12 respectively **(v)** 36 **(vi)** 6.10% **(vii)** 6.90%

Question 9

There were mixed answers to this question, with some candidates obtaining very high marks, but others showing little understanding of the Venn diagrams. With the linear diagram in part **(ii)**, some proceeded well until they reached **(d)**, then returned to **(c)**, changed the answer from 16 to 8, and gave the answer to **(d)** as 16.

Part **(iii)** was well answered, with few multiplying the product of their probabilities by two. A common problem with answers to part **(iv)** was that a region of the diagram presented was unlabelled.

Answers: **(i)(a)** 11 **(b)** 6 **(c)** 4 **(d)** 22 **(e)** 4/15 **(f)** 3/7 **(ii)(a)** 4 **(b)** 9 **(c)** 16 **(d)** 0 **(iii)** 17/75
(iv) fully correct diagram with triple intersection of 1 and double intersections of 5, 2, 0

Question 10

By far the best parts of the question answered were parts **(i)** and **(ii)**. Many correct answers were seen with accurate working. There was much less premature rounding or truncation of decimals this year, and most used the method for standard deviation based on Σfx and Σfx^2, a far better method for computational purposes than that which uses $\Sigma f(x - \text{mean})^2$.

The remaining parts were less well answered, especially part **(v)**, the solution to which required only the product of three simple probabilities. Candidates seem not to have appreciated that there was no probability required for the first digit, which the postman knew, and the next choice was only one out of five. There was also little success in part **(vi)**, even though full marks were possible using an incorrect answer from part **(v)**.

Answers: **(i)** 6–under 9 **(ii)** 7.72, 4.05 **(iii)** km^2 **(iv)** 12, 20 **(v)** 1/60 **(vi)** 59/3600

Question 11

The question on crude and standardised rates continues to be answered exceptionally well, and a good number of fully correct answers was seen. But for part **(vi)** it sometimes seemed as though memorised answers to old questions were being given, with references to, for example, “a healthier environment”, or “chances of a longer life”. The appropriate context should always be considered in the practical application of statistics, and here the context is the hospitals, not the towns.

Answers: **(i)** 69.22 **(ii)** 1.5, 6.25, 80, 155 **(iii)** 64.7875 **(iv)** 68.1725 **(v)** 81 **(vi)** Southshore, because it has the higher standardised mortality rate

STATISTICS

Paper 4040/13

Paper 13

Key Messages

It is sound examination practice to show method clearly, so that marks for method can be awarded even if the answer obtained is incorrect.

After performing a calculation which relates to a specific physical situation, it is worth pausing to consider whether or not the answer obtained is a sensible one. If it is not, the work should be checked to find the error.

General Comments

The overall standard of work showed improvement over that of last year, with some very high marks being obtained, and few very low marks. The best performances in **Section A** were on **Questions 1**, **2** and **4**, and in **Section B** on **Questions 9** and **11**.

Good attention was given to accuracy instructions, where given.

In questions requiring written answers the standard of English was generally good.

Comments on Specific Questions

Section A

Question 1

The first three parts were well answered. A fairly common error in part **(iv)** was to identify the median as the 15.5th item of data rather than the 16th.

Answers: **(i)** 5 **(ii)** 31 **(iii)** 4 **(iv)** 3

Question 2

The first two parts were well answered. Some candidates lost marks in part **(iii)** by not using the squares of the radii. Many did not provide a complete answer to part **(iv)**, offering only the fact that Katrina saw fewer birds than Andy, or that the radius of her chart was smaller than his. For a full answer it was necessary to explain that the consequence of the fewer birds seen by Katrina was that the *proportion* of sparrows seen by her was larger.

Answers: **(i)** 100° **(ii)** 59 **(iii)** 2.41 cm **(iv)** because Katrina counted fewer birds in total, the proportion of sparrows for her was larger

Question 3

Most candidates showed they had a good idea of the nature of a systematic sample in part **(i)**, and there was scarcely any confusion with other types of sampling. But often one or more of the points below, all essential for full marks to be obtained, were omitted. The main focus in many answers was on how many groups the pupils should be divided into and the size of these groups; thus the first and/or third points were frequently ignored.

Good understanding of stratified sampling was shown in part **(ii)**, but very few correct answers were seen in part **(iii)**.

Answers: **(i)** an ordered list of the pupils should be formed from 001 to 175, the first pupil is selected at random from pupils 001 to 007, every 7th pupil thereafter is selected **(ii)** 7 **(iii)** they could have ordered their lists differently / it depends on which numbered pupil they each selected first

Question 4

This question was a good source of marks for many. Any errors were usually in part **(iii)**, when the number of passes was expressed as a percentage of the students taking the course, not the total number of students in the college.

Answers: **(i)** 49 **(ii)** 87.8% **(iii)** 20.7% **(iv)** number of boys increased / number of girls remained fairly constant / overall total of students increased

Question 5

A good number of candidates earned five of the six marks available here. The mark lost was almost always on P(X = 4). Here the incorrect answer of 1/8 was commonly seen, this value corresponding to only one of the two ways in which the selection process as defined could end. It appears not to have occurred to candidates that a check on their answers was possible, the probabilities needing to total to 1.

Answers: **(i)** 3/10 **(iii)** 7/40, 7/24

Question 6

It was clear from many answers to part **(i)** that most candidates did not properly appreciate the significance of the word “variable” in the question. Typical answers to all parts often took the form of “green” or “triangle” or “wood”. Thus confusion was revealed between the value of a variable, and the variable itself. Also, an answer sometimes seen for **(b)** was “the number of pieces in the box”. Whilst this showed understanding of “quantitative and discrete”, it did not show understanding of “variable”. The answers given below are not exhaustive, “shape”, for example, being acceptable for **(i)(a)**.

Answers to part **(ii)** were better, though an answer of 9 was often given in **(a)**.

Answers: **(i)(a)** e.g. colour **(b)** e.g. number of sides **(c)** e.g. mass **(ii)(a)** 11 **(b)** 22 **(c)** 30

Section B

Question 7

Answers to part **(a)** were mixed. Some correct well explained solutions were seen, but others missed out one of the cases, usually the one where the driver has to stop at both sets of lights. No solutions were offered using the complement approach.

Answers to part **(b)** commonly began well, but faded in the later parts. In parts **(iv)** and **(v)** there were commonly too many probabilities in the products needed to obtain the correct answers.

Answers: **(a)** 0.791 **(b)(i)** 0.39 **(ii)** 0.18 **(iii)** 0.43 **(iv)** 0.347 **(v)** 0.489

Question 8

The use of semi-averages to find a line of best fit to experimental data is well understood. Good marks were often obtained on the first three parts. In finding the equation of the line in part **(iv)**, a frequent error was to take the reading of y for a mass of 30 g as c in the equation. Also, accuracy was often lost by using points read from the line drawn in part **(iii)**, when use of the given overall average and calculated semi-averages, through which the line must pass, would have been far more accurate.

For part **(v)** answers had to be in context to earn the mark. Some good answers to the last two parts were seen, very good answers to part **(vii)** making the clear distinction between interpolation and extrapolation.

Answers: **(ii)** (37.5, 31.7), (57.5, 45.7) **(iv)** $y = 0.7x + 5.45$ **(v)** the length of the spring with no mass attached **(vi)(a)** 35 cm **(b)** 58 cm **(vii)** the estimate for 42 g as it is within the range of the measurements obtained; that for 75 g involves extrapolation

Question 9

This question was well answered, with understanding and generally accurate reading of the given graph being shown. Most difficulties were encountered in part **(v)**. The fact that the given information was for the people who had completed the puzzle in the allotted time was almost always ignored, so the 60% was applied incorrectly to 140 instead of 130.

Answers: **(i)** 10 **(ii)(a)** 47s **(b)** 7s **(c)** 33s–33.5s **(iii)(a)** 18 **(b)** 70 **(iv)** 64th **(v)** 70

Question 10

The histogram was generally well understood, with good answers to the first two parts. There were fewer correct answers to part **(iii)**, where some candidates did not seem to be aware that interpolation had to be used. On this question, impossibly large answers were sometimes given. Candidates should always be aware of whether or not their answer is a sensible one for the practical situation of the question. Here, none of the plants is more than 9 cm in height, so if an answer greatly in excess of this was obtained, the candidate should have realised that an error had been made somewhere.

Generally good understanding was shown of the linkage between the answers of parts **(iv)** and **(v).**

Answers: **(i)** 3 cm **(ii)(a)** 5 **(b)** 14 **(iii)** 2.94 cm **(iv)** 7.38 cm, 1.91 cm **(v)** 9.38 cm, 1.91 cm

Question 11

This question was very well done, with many fully correct answers being seen.

Most candidates understood clearly that it was the standardised rate that had to be used to come to a decision in part **(vii)**.

Answers: **(ii)** 4800 **(iii)** 5, 7.5 **(iv)** 4.78 **(v)** 15 **(vi)** 4.86 **(vii)** Birchville, because it has the lower standardised death rate

STATISTICS

Paper 4040/22

Paper 22

Key Messages

This examination requires candidates to be able to calculate statistics, produce statistical diagrams and interpret findings. Candidates scoring the highest marks in the numerical problems provided clear indications of the methods used in logical, clearly presented solutions. The best statistical diagrams were accurately drawn and clearly labelled. In the questions that required written definitions, justification of given techniques, or interpretation, the most successful responses included detailed explanations with, where appropriate, clear consideration of the context of the problem.

General Comments

This examination proved to be a little more difficult than last year's paper. As usual candidates did better on the questions requiring numerical calculations than on those requiring written explanations; in particular, candidates did well in the numerical parts of **Question 10** on price relatives and the early parts of **Question 11** on probability and expectation. There were however two numerical questions that caused difficulty this year, namely **Question 4(ii)** on finding a mean and **Question 6** on linear interpolation. Answers to questions requiring written explanations, such as **Questions 7(a)(iii)** and **10(vii)**, were sometimes too vague or insufficiently detailed; however, in **Question 5(iii)**, for example, there were some good descriptions of the trend and in **Question 7(a)(i)** there were clearly explained advantages and disadvantages of taking a sample. The chart in **Question 3(iii)** was often accurately produced, but a common error was for the axis label to be missing.

Question 11, on probability and expectation, proved to be the least popular of the optional ***section B*** questions, with **Question 7**, on sampling, and **Question 10**, on price relatives, proving to be the most popular.

Comments on Specific Questions

Section A

Question 1

In part **(i)**, most candidates were able to find the mean in degrees Celsius correctly by applying the given formula to the mean given in degrees Fahrenheit. Many candidates, however, applied the same method to obtain the standard deviation, which gave them a negative value. Only a few candidates used the fact that the standard deviation is not affected by the subtraction and thus only multiplied the value given by 5/9. Those candidates who had obtained a negative value for the standard deviation were unable to interpret this figure in part **(ii)**. Most candidates were able to interpret their value for the mean successfully by comparing it with the value given for 2010 and concluded correctly that it was hotter, on average, in 2010 or that there had been an increase in the temperature. Attempts to interpret the standard deviation were often missing and it was rare to see candidates correctly conclude that the temperatures were less varied in 2010.

Answers: **(i)** 16.7 **(ii)** 3.7

Question 2

In part **(a)**, most candidates were able to conclude correctly that, in the final case, where the probability of *A* and the probability of *B* sum to more than one, the two events could not possibly be mutually exclusive. It was more difficult when the sums of the two probabilities were less than one and equal to one. Many candidates correctly realised that there was insufficient information to decide in one of these cases, but it

was rare to see candidates correctly conclude that there was insufficient information to decide in both of these cases. In part **(b)**, many candidates were able to multiply correctly the two probabilities of these independent events to find the probability of the intersection. Most candidates were then able to apply the correct formula to find the probability of the union, with some, however, simply adding the two probabilities and omitting the subtraction of the intersection.

Answers: **(b)(i)** 0.12 **(ii)** 0.58

Question 3

There were many good answers seen to this question. In parts **(i)** and **(ii)**, correct calculations leading to correct results were seen in the vast majority of scripts. In part **(i)**, where the result was given, clearly set out calculations were seen. In part **(iii)**, accurately drawn change charts were often seen. Marks were lost if the axis was not labelled correctly with 'percentage change'. In all statistical diagrams, clear labelling is essential to make the values depicted meaningful.

Answers: **(ii)** 4.3, –1.5

Question 4

The majority of candidates were able to obtain a correct expression for the scaled mark in part **(i)(a)** and evaluate it correctly. In part **(i)(b)**, when candidates obtained a correct expression with two occurrences of the unknown, they were usually able to rearrange it in order to obtain correctly the mark that would remain unchanged by the scaling process. There were, however, a significant number of candidates who began with an incorrect expression containing just one occurrence of the unknown. In part **(ii)**, a large number of candidates tried, incorrectly, to use the sort of approach involving scaling that they had used in the earlier parts of this question. Those that began by multiplying 23 by 56, to find the total of all the boys' marks, usually went on to find a fully correct solution.

Answers: **(i)(a)** 37.5 **(b)** 92.6 **(ii)** 65

Question 5

In part **(i)**, most candidates read the values from the graph accurately and many went on to make the correct adjustment. Common mistakes were not to make any adjustment and, less often, to subtract the seasonal component rather than add it. Most candidates interpreted the trend line correctly in part **(iii)**, but interpreting the seasonal component in part **(ii)** was done less well. In part **(ii)** many candidates tried to relate the quarterly component for quarter III to the other quarters, or suggested that the original values were always exactly \$38 below the trend line, rather than on average below the trend line. In part **(iii)**, most candidates gave a correct description of the general trend by saying that the sales were reducing. Just a few candidates stated incorrectly that the sales were reducing each quarter. Most candidates obtained the correct value for q in part **(iv)**. Mistakes seen included the idea that the seasonal components should add to 1 or that q should be the mean of the other three components.

Answers: **(i)** Quarter I: a value in the range 588–589, quarter II: a value in the range 650–651 **(iv)** 24

Question 6

There were many partially correct solutions seen in both parts of this question, but it was rare to see a fully correct solution. In part **(i)**, there were errors seen with the class boundaries and class widths and some candidates looked for the 45^{th} rather than the 135^{th} value. When the appropriate class was located, the lower class boundary was often taken as 140 rather than 139.5 and the class width as 9 rather than 10. In part **(ii)**, similar errors were seen with the class boundaries, with 6 and 19 appearing in place of 6.5 and 20 in the calculation, and some candidates seemed to confuse frequency values with values of the variable.

Answers: **(i)** 146.7 **(ii)** 34

Section B

Question 7

This question on sampling proved to be the most popular of the ***Section B*** questions. Good answers were often seen in part **(a)(i)**, with many candidates clearly expressing the advantages and disadvantages of taking a sample. The most common advantages seen were that a sample is quicker or cheaper and the most common disadvantage seen was that a sample may not be representative of the whole population. Systematic samples with gaps of 100 were often provided in part **(ii)**, but the correct starting value of 100 was often not seen. In part **(iii)**, some candidates described a representative sample rather than an unbiased sampling method. The best answers described an unbiased sampling method as one in which each member of the population has an equal chance of being selected. It was not sufficient to describe it simply as one which is fair, as this is too vague. Part **(b)(i)** was less structured than some similar questions in the past, with candidates being required to calculate the number of each job type needed as well as to find the sample. It was pleasing therefore to see so many fully correct responses. The most common error was for candidates to try also to stratify by gender. The three things that were required in part **(b)(ii)** were for candidates to identify and total up the genders of the people in their sample, to calculate that the correct ratio should be 4 males and 2 females and to compare these figures to establish that their sample was not representative in terms of gender. Many candidates achieved some, but not all, of the marks in this part; a common misconception being that having equal numbers of males and females meant that the sample was representative, regardless of the ratio in the population. As with a similar question last year, it was good in part **(b)(iii)** to see candidates considering the purpose of a questionnaire when deciding on the most appropriate sampling technique. The best answers stated that job type was more likely than gender to influence how much someone enjoys their work and thus a sample stratified by job type would be the most appropriate.

Answers: **(a)(ii)** 100, 200, 300, 400, 500, 600 **(b)(i)** 24, 19, 50, 43, 38, 13.

Question 8

Most candidates correctly stated, in part **(a)(i)**, that 'the options chosen by the students' was a qualitative variable as the values of the variable are not numerical or are described using words. A few candidates gave the wrong answer and referred to the number of options. In part **(a)(ii)**, most candidates correctly identified that the 33 students were represented by 22% in the percentage sectional bar chart. There are alternative correct approaches that can be taken, but most candidates went on to find the number of students studying each of the other combinations. Some candidates were unable to proceed beyond this point or incorrectly halved these figures before proceeding. Many candidates then correctly added appropriate numbers of students together from the relevant combinations for each subject. It was important in this question for candidates to communicate carefully the methods that they were using, so that part marks could be awarded for partially correct working. In part **(a)(iii)**, candidates needed to appreciate that, although numbers are not known for 2013, it is still possible to make comparisons between subject areas within that year, but that comparisons with 2012 are not possible. In part **(a)(iii)(a)**, most candidates who got the answer correct stated that the assertion was definitely false, because 46% studied Plumbing whilst 80% studied Carpentry. Alternative correct comparisons could be made, such as to compare the percentage studying Plumbing and Building with the percentage studying Carpentry and Building. Many candidates however incorrectly stated that there was insufficient information to decide, because numbers were not known. In part **(a)(iii)(b)**, the fact that the numbers in 2013 were unknown meant that there was insufficient information to decide. Many candidates, however, incorrectly stated that the assertion was correct, comparing the percentages in the sectional bar chart. In part **(b)(i)**, many candidates correctly stated that the variable 'the distance travelled by each cyclist' was continuous, but the reasons given were not always correct. In the best answers, candidates said that the distance travelled can take any value in a range or that the distance travelled is a quantity that is measured. It is not sufficient to say that the distance travelled can take a decimal value. The class boundaries in part **(b)(ii)** were often incorrect, with 22.5 and 25.5 being the most common incorrect answers. Boundaries such as these, that did not leave gaps between the classes, often therefore led to a correct value for the class width. Candidates should take care when interpreting frequency polygons, as the comparisons made often appeared to be based on the heights of the polygons, with an incorrect answer of 'pedal powers cycled further' rather than 'speedy wheelers cycled further' often being seen in part **(b)(iii)**.

Answers: **(a)(ii)** 54, 129, 117 **(b)(ii)** 23, 26, 3

Question 9

Very few candidates scored full marks on this question, with parts **(iii)** and **(iv)** proving particularly difficult for many. Parts **(i)(a)** and **(i)(b)** were usually completed correctly, but sometimes an incorrect denominator of '18' appeared for the conditional probability in part **(i)(c)**. Part **(ii)** was often completed correctly, with the most common mistake being either to miss the multiplication by 2 or to have 18 × 18 in the denominator. Many attempts at part **(iii)** involved the addition of several probability products, rather than the one required product of four probabilities. Some of the attempts that were of the correct format often had the correct denominator but an incorrect numerator. In part **(iv)**, correct or almost correct values were often seen in the numerators, but often the denominators were just 18 × 17 × 16 × 15. Sometimes the products of four probabilities were multiplied by 4 or 6 rather than 2. Many candidates recovered to find the numerator correctly in part **(v)** and this was often seen with a correct denominator, but sometimes with other denominators. Other common wrong answers involved multiplying together the 1/3 and 2/5.

Answers: **(i)(a)** 4/9 **(b)** 2/9 **(c)** 3/8 **(ii)** 77/153 **(iii)** 91/816 **(iv)** 31/84 **(v)** 4/11

Question 10

Most candidates provided sufficient clear working to show that the price relative given in part **(i)** was correct. Many correct answers were seen for part **(ii)**, with the most common error being the multiplication of 103 by the 2012 rate of pay, rather than the base year, 2011, rate of pay. In part **(iii)**, it was important to state that the price relative of 97 indicated that the 'price' or 'cost' (and not the 'expenditure') of veterinary services had decreased by 3%. It was also important to state that this change had taken place between 2011 (the base year) and 2012. Often one of these points was either incorrect or missing. Many candidates successfully found the price relatives in part **(iv)**, with just a few candidates using subtraction rather than division in their calculations. The weighted aggregate cost index was usually found correctly in part **(v)**, with just a few candidates using the wrong base year for their calculations. Calculations seen in part **(vi)** were usually correct, with a few candidates struggling, or omitting, to give their answer correct to three significant figures. It is important that candidates appreciate that the result of such a calculation is only an estimate, and therefore any greater degree of accuracy is not required. The potential causes of inaccuracy in such an estimate were the subject of part **(vii)**. Many candidates did not state the correct answer that any changes in the price of animal food **(A)** or the price of fuel **(D)** would have been accounted for within the price relatives, and thus they were not possible explanations for the estimate being inaccurate.

Answers: **(ii)** 8.20 **(iv)** 106, 96, 97, 107 **(v)** 101.4–104.7 **(vi)** 323 000

Question 11

This was the least popular of the optional questions, although those that attempted it were often successful, particularly with the earlier parts. Most candidates correctly found the value of p in part **(i)**. Many correct calculations for the expected profit or loss were seen in part **(ii)**, with the most common error being the omission of the fact that the answer represented a 'loss'. In part **(iii)(a)**, some candidates assumed that the probabilities were no longer as in the table and had become equal probabilities. Many correct solutions to this part were, however, seen and candidates who had the correct structure to their answer to part **(iii)(a)** often went on to complete parts **(iii)(b)** and **(iii)(c)** in the correct way. Part **(iv)** proved to be the most difficult part of this question. Most candidates who attempted it found the correct probabilities for the arrow to land in each sector. It was, however, rare to see the probabilities multiplied by x, $2x$ and $3x$ respectively and set equal to 11; more commonly each probability was multiplied by x and set equal to 11 or simply each probability was multiplied by 11.

Answers: **(i)** 0.05 **(ii)** loss of 0.15 **(iii)(a)** 0.32 **(b)** 7.50 **(c)** loss of $48 **(iv)** 6, 12, 18

STATISTICS

Paper 4040/23

Paper 23

Key Messages

This examination requires candidates to be able to calculate statistics, produce statistical diagrams and interpret findings. Candidates scoring the highest marks in the numerical problems provided clear indications of the methods used in logical, clearly presented solutions. The best statistical diagrams were accurately drawn and clearly labelled. In the questions that required written definitions, justification of given techniques or interpretation, the most successful responses included detailed explanations with, where appropriate, clear consideration of the context of the problem.

General Comments

This examination proved to be a little more difficult than last year's paper. As usual candidates did better on the questions requiring numerical calculations than on those requiring written explanations; in particular, candidates did well in **Question 2**, on scaling, and parts of the probability in **Questions 7** and the early parts of **Question 11**. There were however two numerical questions that caused difficulty this year, namely **Question 6** on mean and variance and parts of **Question 8** on linear interpolation. Answers to questions requiring written explanations, such as **Questions 8(v)** and **9(v)**, were sometimes too vague or insufficiently detailed; however, in **Question 5(a)** there were clearly explained advantages and disadvantages of taking a sample. The chart in **Question 4(i)** was often accurately produced, but a common error was for the vertical axis label to be missing.

Question 11, on probability and expectation, proved to be the least popular of the optional ***Section B*** questions, with **Question 8**, on linear interpolation, and **Question 10**, on moving averages, proving to be the most popular.

Comments on Specific Questions

Section A

Question 1

Candidates found obtaining the class boundaries in part **(i)** difficult, with boundaries of 8.5 and 11.5 being a common error, but also boundaries such as 9 and 11, without a difference of 3, being seen. Most candidates were able to draw a pair of labelled frequency polygons in part **(ii).** Errors were sometimes seen in the vertical heights of the plots with some candidates not consistently using 2 small squares for each child. A more common error was for candidates to use their lower or upper class boundaries rather than the midpoints for the horizontal position of the plots. Most candidates were able to correctly interpret the frequency polygons in part **(iii)** to observe that the children at Aqua Splash were older. In order to score the mark candidates needed to make a general comment rather than simply to compare the number children for any one specific age group.

Answers: **(i)** 9, 12

Question 2

Many candidates answered this question correctly, with parts **(i)** and **(iii)** being particularly well done. Any errors seen in these two parts tended to be errors in the algebraic manipulation, with correct initial expressions in the unknowns being provided. Most candidates also answered part **(ii)** correctly, although many found the scaled mark for Physics/Chemistry first, rather than solving the problem directly, and this sometimes led either to errors in the algebraic manipulation or to some candidates appearing to forget to complete the problem, having found the scaled mark.

Answers: **(i)** 59 **(ii)** 96 **(iii)** 7.6

Question 3

Most candidates realised that the first step in part **(a)(i)** was to find the probability of *A*. A common error was for this to be found by subtracting the probability of *B* from the probability of not *A*. Candidates who had made this error sometimes went on to use a correct expression for the probability of the intersection. More errors were made in part **(a)(ii)**, with only a minority of candidates attempting to multiply their probability of *A* by the probability of *B* and to compare this result to their answer to part **(a)(i)**. In part **(b)**, two correct pairs were often seen, but some candidates gave just one of the correct pairs and some candidates listed all the pairs that were not mutually exclusive.

Answers: **(a)(i)** 0.2 **(ii)** $0.3 \times 0.6 \neq 0.2$ so not independent **(b)** *C* and *D*, *D* and *F*

Question 4

Many accurate percentage sectional bar charts were seen in part **(i)**, with candidates correctly calculating the required percentages and then plotting these values carefully using sensible scales. Bars were usually labelled or a key was provided but some vertical scales were left unlabelled. In all statistical diagrams clear labelling is essential to make the values depicted meaningful. In part **(ii)**, the majority of candidates correctly stated that country *A* had the greater area of urban land, but that this area represented a smaller proportion of the total area than for country *B*. The most common error was for candidates to make two statements using the percentage section bar chart, comparing the various types of land, rather than comparing the area of urban land in the two countries, using both the table and the chart, as instructed.

Answers: **(i)** 9, 35, 56; 14, 47, 39

Question 5

Good answers were often seen in part **(a)**, with many candidates clearly expressing the advantages and disadvantages of taking a sample. The most common advantages seen were that a sample is quicker or cheaper and the most common disadvantage seen was that a sample may not be representative of the whole population. Part **(b)** proved to be more difficult, although it was good to see many candidates correctly explain in part **(b)(i)** that the given statement was sometimes true and that it would depend on the proportions of males and females in the original population from which the sample had been taken. Some candidates, however, incorrectly wrote that it was always true, giving as a reason that equal numbers of males and females avoided bias. In part **(b)(ii)** it was rare to see candidates correctly explain that the statement was sometimes true and that it depends on whether the list itself is random or whether there is some order to the list. Most candidates thought that the statement was always true.

Question 6

Many fully correct answers were seen in part **(i)**, although it was not uncommon for candidates to find the total number of visitors for all seven days of the week without subtracting the number of visitors from Monday to Saturday. The calculation of the variance, in part **(ii)**, proved much more difficult with some candidates leaving this part blank. Some candidates seemed to confuse variance with standard deviation, and others who correctly used the formula for variance to find the sum of the number of days squared for the first six days then went on to incorrectly square root this value.

Answers: **(i)** 45 **(ii)** 66.9

Section B

Question 7

Candidates generally did well on this probability question. Fully correct answers were often seen for part **(i)**, with the most common errors being either candidates treating the problem as if the two beads were being selected from the same bag, without replacement, or omitting to consider that the beads could be selected in any order. Part **(ii)** proved to be more difficult, although many fully correct solutions were seen. The most common error was for candidates to find the probability that exactly two black beads were selected rather than at least two black beads. Part **(iii)** was well done with almost all candidates correctly dealing with the fact that the beads were not being replaced. A few candidates did not consider that there were three ways to select the two white beads. In part **(iv)** many correct solutions were seen, although some candidates thought that the beads could alternate in colour in more than one way, and some candidates seemed to misunderstand the phrase 'alternate in colour'.

Answers: **(i)** 12/25 **(ii)** 81/125 **(iii)** 3/10 **(iv)** 1/10

Question 8

This was a popular ***Section B***, question although some weaker candidates found it difficult, particularly the last three parts. Most candidates were able to find the cumulative frequencies in part **(i)** and the median in part **(ii)**. In part **(iii)** it was quite common to see candidates calculating a value of p for which 20% of the residents were aged less than p years, rather than p years or more. Similarly, in part **(iv)** candidates were often able to use a correct method to find the number of people aged either less than 18 or less than 67 but were not able to use a correct method to find the number aged between 18 and 67. Some candidates who correctly found this number forgot to convert it to a percentage for the final mark. In part **(v)** more able candidates could often explain correctly that the answers are estimates because the data is grouped. It was, however, rare to see the comment that in linear interpolation the data is assumed to be evenly distributed within each class. Some weaker candidates made comments referring to rounding as the reason that the answers were only estimates or made comments that were too vague about linear interpolation only producing estimates, without giving reasons.

Answers: **(i)** 16, 37, 70, 92, 108, 116, 120 **(ii)** 27.0 **(iii)** 45 **(iv)** 65%

Question 9

The missing values from the tables were usually correctly inserted in parts **(i)** and **(ii)**. Candidates found part **(iii)** difficult, with some omitting an attempt at a calculation to show the given result for the weights. Any attempt to multiply the number of workers at each grade by the rate of pay for that grade was needed, followed by division to produce the given weights. It was rare to see candidates explain that the assumption made was that each worker does the same number of hours. The weighted aggregate cost index was usually correctly calculated in part **(iv)**, although some candidates did omit this part of the question. Part **(v)** was also omitted by some candidates, with others answering either the first part of the question or the second part but not both. In order to score full marks candidates needed firstly to state that their answer indicated that the total wage bill had increased by 3% between 2011 and 2013, and secondly that the assumption was that there is no change in the number of workers at each grade or in the number of hours that they work.

Answers: **(i)** 7.35, 7.95 **(ii)** 100s in first column, 105, 103, 97, 97 **(iv)** 103

Question 10

In part **(i)** most candidates were able to give a correct reason for finding moving average values and explain why it is useful to do this: namely to remove variation in order to find the trend or to find the trend in order to make predictions. Some candidates only mentioned finding the trend, and some misunderstood the question and discussed the need for centring. In part **(ii)** a clear explanation was required, and thus for full marks it was necessary to state that centring was needed to make the moving average values coincide with original data items. Statements, which were commonly seen, such as "centring is necessary because the number of quarters in a year is even" or "because four is an even number" were enough to score part-marks. Most candidates were able to find the correct values for a, b and c in part **(iii)**. Unfortunately, there was a typographical error in the sixth value for the four-quarter total: 228.8 should have read 228.2; candidates who used this value correctly to find a or b were awarded full marks. It was good to see many candidates correctly finding the estimate for the seasonal component in part **(iv)**. Some partially correct solutions were also seen where candidates demonstrated an understanding that the difference between the centred moving

average value and the corresponding number of marriages would be required. A few candidates left this part of the question blank. Points were accurately plotted by the majority of candidates in part **(v)**, with suitable trend lines passing through the points. Comments on the trend line in part **(vi)** often correctly described the general trend, but were not always in the context of the question. Candidates who had made a good attempt at the seasonal component were often able to use their graph together with their value for the seasonal component to make the estimate required in part **(vii)** – although many left the answer in thousands, without expressing them as such; so estimates such as 62.1 marriages in quarter II of 2014, rather than 62 100 or 62.1 thousand, were seen. Others, however, simply took a reading from the graph without making the required adjustment.

Answers: **(iii)** 95.9, 226.2, 58.2 **(iv)** 6.7 thousand **(vii)** 61 900 to 62 300

Question 11

This question on probability and expectation was the least popular of the ***Section B*** questions. Many fully correct responses were seen for part **(i)(a)**, with only a few candidates omitting to state that the expected value represented a loss. In part **(i)(b)(i)** most candidates correctly found the probability of scoring a double, although some omitted to include a double three. Some candidates then incorrectly multiplied by two (the cost of the game), rather than multiplying by the unknown and setting equal to two. Some candidates did not begin their solution to part **(i)(b)(ii)** by finding the number of people expected to win, as instructed in the question. Those that used the probability of getting a double from the previous part and multiplied it by 90 people often went on to produce fully correct solutions. Only a few candidates were successful with part **(ii)**, the most common error being to multiply the probabilities of getting one spot and two spots each by x, rather than by x and $2x$ respectively, prior to adding these products and equating to two.

Answers: **(i)(a)** loss of 0.33 **(b)(i)** $5 **(b)(ii)** $5 profit **(ii)** $1.50, $3